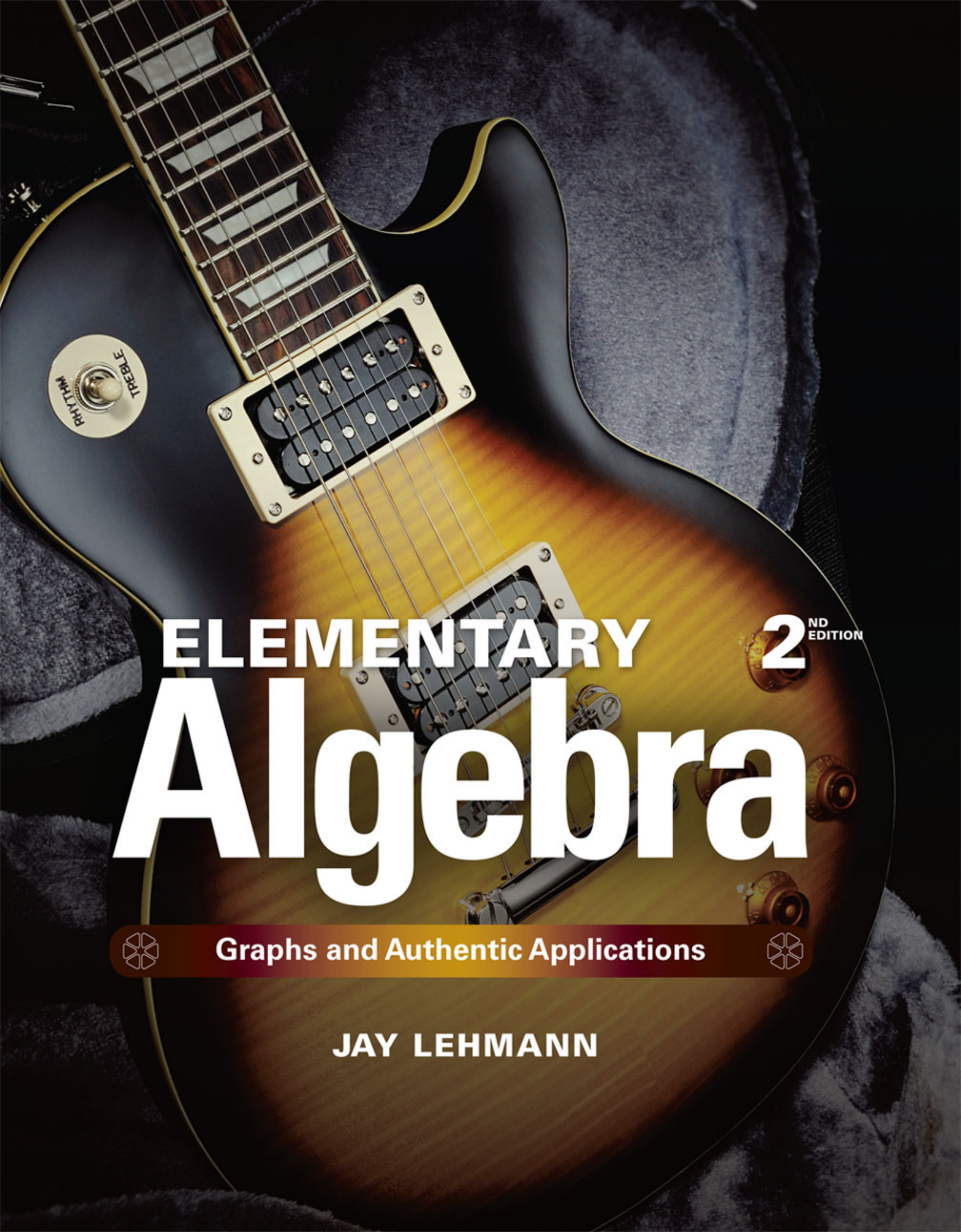

ELEMENTARY Algebra

2ND EDITION

Graphs and Authentic Applications

JAY LEHMANN

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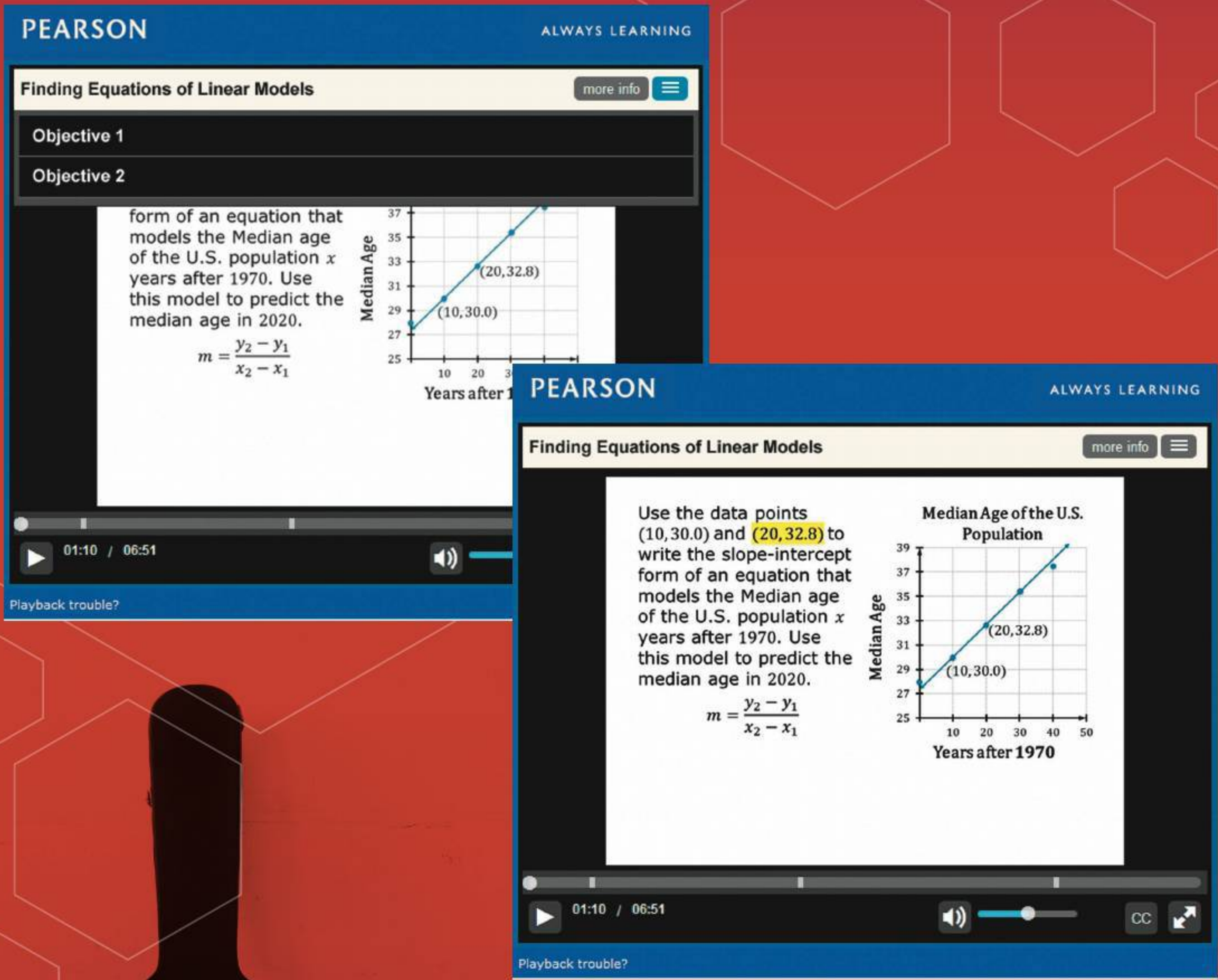

Students can make the most of their study time by preparing for exams with the new Interactive Lecture Series.

Interactive Lectures highlight key examples and exercises from every section of the textbook. A new interface allows easy navigation to sections, objectives, and examples. These videos are available in MyMathLab.

Jay Says...

Before writing my algebra series, it was apparent that my students couldn't relate to the applications in the course; they would repeatedly ask, "What is this good for?" To try to bridge that gap, I wrote some labs, which helped my students collect data, find models via curve fitting, and use the models to make estimates and predictions. My students really loved working with the current, compelling, and authentic data and experiencing how mathematics truly is useful.

My students' response was so strong that I decided to write an algebra series. Little did I know that, to realize this goal, I'd need to embark on a 15-year challenging journey, but the rewards of hearing such excitement from students and faculty across the country have made it all worthwhile! I'm proud to have played even a small role in raising people's respect and enthusiasm for mathematics.

I've tried to honor my inspiration: By working with authentic data, students can experience the power of mathematics. A random-sample study at my college suggests that I'm achieving this goal. The study concludes that students who used my series were more likely to feel that mathematics would be useful in their lives $(p = 0.0061)$ as well as in their careers $(p = 0.024)$.

In addition to curve fitting, my approach includes other types of meaningful modeling, directed-discovery explorations, conceptual questions, and, of course, a large bank of skill problems. The curve-fitting applications serve as a portal for students to see the usefulness of mathematics so that they become fully engaged in the class. Once involved, they're more receptive to all aspects of the course.

Elementary Algebra

Graphs & Authentic Applications

Second Edition

Jay Lehmann
College of San Mateo

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To Dylan, for your curiosity and wonder of mathematics.

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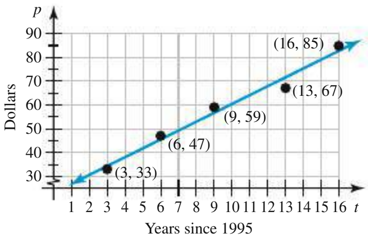

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1988	57.4
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1980	26.6
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2010	48.4

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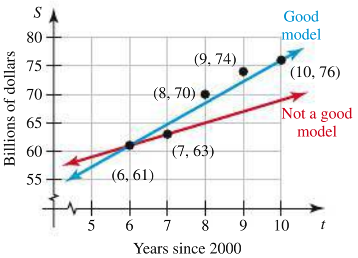

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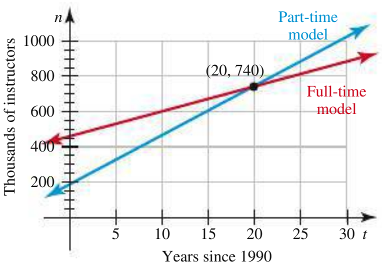

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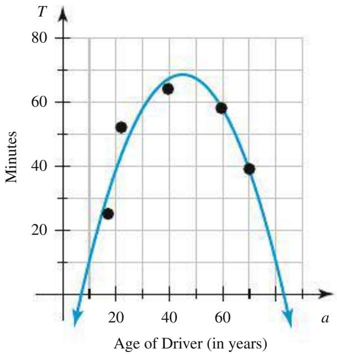

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2011	72.3

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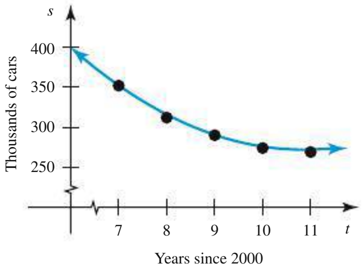

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2002	27.3	109.3
2004	41.3	112.0
2006	58.6	114.4
2008	67.4	116.8
2010	72.5	117.5

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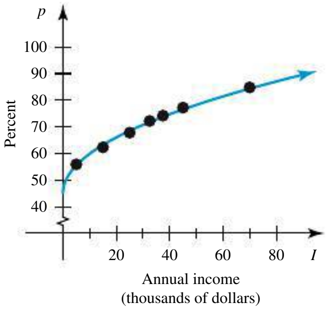

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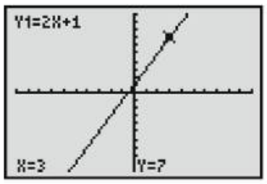


Preface

"The question of common sense is always, 'What is it good for?'—a question which would abolish the rose and be answered triumphantly by the cabbage."

—James Russell Lowell

These words seem to suggest that poet and editor James Russell Lowell (1819–1891) took elementary algebra. How many times have your students asked, "What is it good for?" After years of responding "You'll find out in the next course," I began a quest to develop a more satisfying and substantial response to my students' query.

APPROACH AND ORGANIZATION

Curve-Fitting Approach Although there are many ways to center an elementary algebra course around authentic applications, I chose a curve-fitting approach for several reasons. A curve-fitting approach

- allows great flexibility in choosing interesting, authentic, current situations to model.
- emphasizes key concepts and skills in a natural, substantial way.
- deepens students' understanding of equations in two variables, because it requires students to describe these equations graphically, numerically, symbolically, and verbally.
- unifies the many diverse topics of a typical elementary algebra course.

To fit a curve to data, students learn the following four-step modeling process:

1. Examine the data set to determine which type of model, if any, to use.
2. Find an equation of the model.
3. Verify that the model fits the data.
4. Use the model to make estimates and predictions.

This four-step process weaves together topics that are crucial to the course. Students must notice numerical patterns from data displayed in tables, recognize graphical patterns in scattergrams, find equations of models, graph models, and solve equations.

Not only does curve fitting foster cohesiveness within chapters, but it also creates a parallel theme for each set of chapters that introduces and discusses a new type of model. This structure enhances students' abilities to observe similarities and differences among fundamental models such as linear models, quadratic models, rational models, and radical models.

Unique Organization Many college students who take elementary algebra had significant difficulties with the equivalent course in high school. These students face a greater challenge in the college course, because they must complete the course in one semester, rather than in two. Instead of presenting the material in the "same old way," this text provides a unique organization that will better aid students in succeeding.

The text uses modeling to provide the "big picture" before going into details. For example, Chapter 1 gives an overview of linear modeling, which is the main theme of Chapters 1–6, and Section 7.2 provides an overview of quadratic modeling, which is a major focus of Chapters 7–9. Using modeling to provide the big picture not only is good pedagogy, but also sets the tone that this course will be different, interesting, alive, and relevant, inviting students' creativity into the classroom.

The organization of the text is also unique in that students perform graphing (in Chapter 3) before they manipulate expressions and equations (in most of Chapters 4–11). That way, students can visualize the mathematics early on, making them better able to solve equations in one variable graphically. This early-graphing organization also buys students a bit more time to find their "sea legs" before moving on to the more challenging manipulation work.

Several sections discuss how to use graphs and tables to solve equations in one variable.

NEW TO THE SECOND EDITION

Students will benefit from the following changes to the second edition of *Elementary Algebra: Graphs and Authentic Applications:*

- 149 data sets in examples and exercises have been replaced with more compelling and current topics.
- 175 data sets in examples and exercises have been augmented to include values for recent years.
- 55 new data sets have been added to address new concepts.
- All eight Climate Change labs have been updated.
- 100 conceptual exercises have been added.
- Percent applications have been added to Sections 2.6, 4.3, and 4.4.
- Sections 3.2 and 3.5 now contain graph-related modeling exercises.
- Additional examples and exercises addressing the commutative and associative laws have been added to Section 4.1.
- Modeling exercises that require students to define variables have been added to Sections 4.3, 4.4, 5.3, and 5.4.
- Authentic applications that do not involve curve fitting have been added to Sections 4.4, 5.3, 5.4, 7.6, and 7.7.
- The number of exercises addressing exponential properties has been increased in Sections 7.3 and 7.4.
- Section 7.8 on dividing polynomials has been added.
- Grids have been added to most graphs of models so students can better line up inputs and outputs.
- New Interactive Video Lecture Series are available in MyMathLab.
- The number of exercises available in MyMathLab/MathXL has been increased.

CONTINUED FROM THE FIRST EDITION

HOMEWORK SETS

The homework sets have been carefully structured so the exercises are well paired and progress gradually in difficulty. Students receive ample opportunities to master both procedures and concepts. In addition, two special types of exercises are included to help students succeed in elementary algebra and prepare them for their next course:

Related Review Exercises These exercises relate current concepts to previously learned concepts. The exercises occur near the end of all Homework sections in Chapters 5–11 and serve as constant reinforcement, helping students draw connections between previously learned and new material.

Expressions, Equations, and Graphs These exercises are designed to help students gain a solid understanding of those core concepts, including how to distinguish among them. The exercises are included at the end of all Homework sections in Chapters 5–11, but their foundation is laid in Section 4.5, which is devoted to making such distinctions.

BUILDING STUDY SKILLS

The following features have been included throughout the text to help students improve their study skills, to motivate students, and to provide just-in-time support.

Tips for Success Many sections close with tips for success in mathematics. These tips are intended to help students succeed in the course. A complete listing is included in the index.

Warnings These are discussions (flagged by the margin entry "WARNING") that address students' common misunderstandings about key concepts and help the students avoid such misunderstandings.

Chapter Opener Each chapter begins with a description of an authentic situation that can be modeled by the concepts discussed in the chapter.

TECHNOLOGY, EXPLORATIONS, AND LABS

Technology The text assumes students have access to technology, such as the TI-83 or TI-84 graphing calculator. Technology of this sort allows students to create scattergrams and check the fit of a model quickly and accurately. It also empowers students to verify their results from Homework exercises and efficiently explore mathematical concepts in the Group Explorations.

The text supports instructors in holding students accountable for all aspects of the course without the aid of technology, including finding equations of linear models. (Linear regression equations are included in the Answers section, because it can be difficult or impossible to anticipate which points a student will choose in trying to find a reasonable equation.)

Appendix A: Using a TI-83 or TI-84 Graphing Calculator Appendix A contains step-by-step instructions for using the TI-83 and TI-84 graphing calculators. A subset of this appendix can serve as a tutorial early in the course. In addition, when the text requires a new calculator skill, students are referred to the appropriate section in Appendix A.

Group Explorations All sections of the text contain one or two explorations that support student investigation of a concept. Instructors can use explorations as collaborative activities during class time or as part of homework assignments. Some explorations lead students to think about concepts introduced in the current section. Other, "Looking Ahead" explorations are directed-discovery activities that introduce key concepts to be discussed in the section that follows. The explorations empower students to become active explorers of mathematics and can open the door to the wonder and beauty of the subject.

Taking it to the Lab Laboratory assignments have been included at the end of most chapters, to increase students' understanding of concepts and the scientific method. These labs reinforce the idea that mathematics is useful. They are also an excellent avenue for more in-depth writing assignments.

Some of the labs are about climate change and have been written at a higher reading level than the rest of the text in order to give students a sense of what it is like to perform research. Students will find that by carefully reading (and possibly rereading) the background information, they can comprehend the information and apply concepts they have learned in the course to make estimates and predictions about this compelling, current, and authentic situation.

RESOURCES FOR INSTRUCTORS

Instructor's Resource Manual This manual contains suggestions for pacing the course and creating homework assignments. It discusses how to incorporate technology and how to structure lab and project assignments. The manual also contains section-by-section suggestions for presenting lectures and for undertaking the explorations in the text.

Instructor's Solutions Manual This manual includes complete solutions to the even-numbered exercises in the homework sections of the text.

MyMathLab® Online Course (access code required) MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy to use format. MyMathLab delivers **proven results** in helping individual students succeed. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit www.mymathlab.com or contact your Pearson representative.

New Ready to Go courses provide students with all the same great MyMathLab features, but make it easier for instructors to get started. Both the Standard and Ready To Go courses include pre-made homework and quizzes, which are pre-assigned in the Ready To Go course to make creating a course even easier.

TestGen TestGen enables instructors to build, edit, print, and administer tests by using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple, but equivalent, versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. Tests can be printed or administered online. The software and testbank are available for download from Pearson Education's online catalogue.

PowerPoint Lecture Slides (download only) Available through www.pearsonhighered .com or inside your MyMathLab course, these fully editable lecture slides include definitions, key concepts, and examples for use in a lecture setting and are available for each section of the text.

RESOURCES FOR STUDENTS

New Interactive Video Lecture Series This series has been completely revised to provide students with extra help for each section of the textbook. The Lecture Series includes:

- Interactive Lectures that highlight key examples and exercises for every section of the textbook.
- A new interface that allows easy navigation to sections, objectives, and examples.

These lectures are available in MyMathLab.

Student Solutions Manual This manual contains the complete solutions to the odd-numbered exercises in the Homework sections of the text.

GETTING IN TOUCH

I would love to hear from you and would greatly appreciate receiving your comments regarding this text. If you have any questions, please ask them, and I will respond.

Thank you for your interest in preserving the rose.

Jay Lehmann
MathNerdJay@aol.com

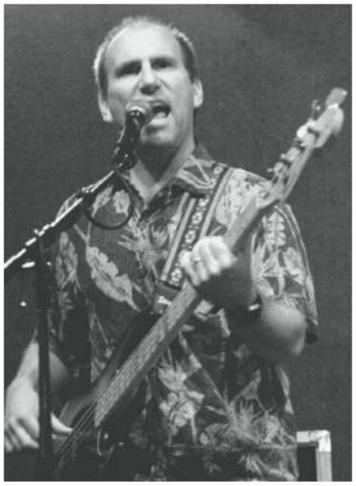
To the Student

You are about to embark on an exciting journey. In this course, you will not only learn more about algebra but also how to apply it to describe and make predictions about authentic situations. This text contains data that describe hundreds of situations. Most of the data have been collected from recent newspapers and Internet postings, so the information is current and of interest to the general public. I hope that includes you.

Working with authentic data will make mathematics more meaningful. While working with data about authentic situations, you will learn the meaning of mathematical concepts. As a result, the concepts will be easier to learn, because they will be connected to familiar contexts. And you will see that almost any situation can be viewed mathematically. That vision will help you understand the situation and make estimates and/or predictions.

Many of the problems you will explore in this course involve data collected in a scientific experiment, survey, or census. The practical way to deal with such data sets is to use technology. So, a graphing calculator or computer system is required.

Hands-on explorations are rewarding and fun. This text contains explorations with step-by-step instructions that will lead you to *discover* concepts, rather than hear or read about them. Because discovering a concept is exciting, it is more likely to leave a lasting impression on you. Also, as you progress through the explorations, your ability to make intuitive leaps will improve, as will your confidence in doing mathematics. Over the years, students have remarked to me time and time again that they never dreamed that learning math could be so much fun.



Jay has a variety of interests. He is pictured here playing with his rock band, The Procrastinistas. (Photo courtesy of Rick Gilbert)

This text contains special features to help you succeed. Many sections contain a Tips for Success feature. These tips are meant to inspire you to try new strategies to help you succeed in this course and future courses. Some tips might remind you of strategies you have used successfully in the past but have forgotten. If you browse through all of the tips early in the course, you can take advantage of as many of them as you wish. Then, as you progress through the text, you'll be reminded of your favorite strategies. A complete listing of Tips for Success is included in the index.

Other special features that are designed to support you in the course include Warnings, which can help you avoid common misunderstandings; Key Points summaries, which can help you review and retain concepts and skills addressed in the chapter you have just read; Related Review exercises, which can help you understand current concepts in the context of previously learned concepts; and Expressions, Equations, and Graphs exercises, which can help you understand and distinguish among these three core concepts.

Feel free to contact me. It is my pleasure to read and respond to e-mails from students who are using my text. If you have any questions or comments about the text, feel free to contact me through e-mail or the website www.pearsonhighered.com/lehmannseries.

Jay Lehmann
MathNerdJay@aol.com

Acknowledgments

Writing a modeling textbook is an endurance run I could not have completed without the dedicated assistance of many people. First, I am greatly indebted to Keri, my wife, who yet again served as an irreplaceable sounding board for the multitude of decisions that went into creating this text. In particular, I credit her internal divining rod in selecting captivating data from a mound of data sets I have collected.

I have received much support from the following professors: Ken Brown, Gary Church, Jon Freedman, Eric Freidenreich, Jenny Freidenreich, Cheryl Gregory, Rick Hough, Evan Innerst, and Tadashi Tsuchida. Over the years, they have given much sound advice in responding to my countless e-mail inquiries.

I acknowledge several key people at Pearson Education. I'm very grateful to Editor-in-Chief Michael Hirsch, who has shared in my vision for this textbook and has made significant investments to make that vision happen. The book has been greatly enhanced through the support of Senior Acquisitions Editor Dawn Giovanniello, who has made a multitude of contributions, including assembling an incredible team to develop and produce this textbook. The team includes Senior Content Editor Lauren Morse, who handled countless tasks to support me in preparing the manuscript for production, leading to a significantly better book.

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I also thank these reviewers, whose thoughtful, detailed comments helped me sculpt this text into its current form:

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Index of Applications

F

V

W

Introduction to Modeling

1

Think about the last concert you attended. What was the ticket price? Was it worth it? The average ticket price for the top-50-grossing concert tours has increased greatly (see Table 1). In Example 2 of Section 1.4, we will predict when the average ticket price will be $105.

In this course, we will discuss how to describe the relationship between two quantities that occur in an authentic situation. For example, we will describe how the average ticket price for the top-50-grossing concert tours has changed over time. In Chapters 1–6, we will focus on how to use (straight) lines to describe authentic situations. In Chapters 7–11, we will discuss other types of *curves* that can be used to describe authentic situations.

Table 1 Average Ticket Prices for Top-50-Grossing Concert Tours

Year	Average Ticket Price (dollars)
1998	33
2001	47
2004	59
2008	67
2011	85

Source: *Pollstar*

1.1 Variables and Constants

Objectives

- Know the meaning of *variable* and *constant*.
- Know the meaning of *counting numbers*, *integers*, *rational numbers*, *irrational numbers*, *real numbers*, *positive numbers*, and *negative numbers*.
- Use a number line to describe numbers.
- Graph data.
- Find the average (or mean) of a group of numbers.
- Know how to describe a concept or procedure.

In this section, we will work with *variables* and *constants*, two extremely important building blocks of algebra. We will also discuss various types of numbers and how to describe numbers visually.

Variables

In arithmetic, we work with numbers. In algebra, we work with *variables* as well as numbers.

Definition Variable

A **variable** is a symbol that represents a quantity that can vary.

For example, we can define h to be the height (in feet) of a specific child. Height is a quantity that varies: As time passes, the child's height will increase. So, h is a variable. When we say $h = 4$, we mean the child's height is 4 feet.

We will discuss other roles of a variable in Sections 2.1 and 4.3.

Example 1 Using a Variable to Represent a Quantity

1. Let s be a car's speed (in miles per hour). What is the meaning of $s = 60$?
2. Let n be the number of people (in millions) who work from home at least once a week during normal business hours. For the year 2010, $n = 22$ (Source: *World at Work*). What does that mean in this situation?
3. Let t be the number of years since 2010. What is the meaning of $t = 4$?

Solution

1. The speed of the car is 60 miles per hour.
2. In 2010, 22 million people worked from home at least once a week during normal business hours.
3. $2010 + 4 = 2014$; so, $t = 4$ represents the year 2014.

There are many benefits to using variables. For example, in Problem 2 of Example 1, we found that the simple equation "$n = 22$" means the same thing as the wordy sentence "22 million people worked from home at least once a week during normal business hours." Variables can help us describe some situations with a small amount of writing.

In Problem 3 of Example 1, we described the year 2014 by using $t = 4$. So, our definition of t allows us to use smaller numbers to describe various years—an approach that will be helpful throughout the course.

We will see other benefits of variables as we proceed through the course.

Example 2 Using a Variable to Represent a Quantity

Choose a symbol to represent the given quantity. Explain why the symbol is a variable. Give two numbers that the variable can represent and two numbers that it cannot represent.

1. the weight (in pounds) of a baby at birth
2. the number of people who live in a two-bedroom house

Solution

1. Let w be the weight (in pounds) of a baby at birth. The weight of a baby at birth can vary, so w is a variable. For example, w can represent the numbers 6 and 8, because babies can weigh 6 or 8 pounds at birth. The variable w does not represent 0 or 300, because babies cannot weigh 0 or 300 pounds at birth!
2. Let n be the number of people who live in a two-bedroom house. The number of people who live in a two-bedroom house can vary, so n is a variable. For example, n can represent the numbers 2 and 3, because 2 or 3 people can live in a two-bedroom house. The variable n cannot represent the numbers 5000 or $\frac{1}{2}$, because 5000 people cannot live in a two-bedroom house and half of a person doesn't make sense.

In Problem 1 of Example 2, we stated that the units of w are pounds. Without stating the units of w, "$w = 10$" could mean the baby's weight was 10 ounces, 10 pounds, or 10 tons! In defining a variable, it is important to describe the variable's units.

Constants

A variable is a symbol that represents a quantity that can vary. When we use a symbol to represent a quantity that does *not* vary, we call that symbol a *constant*. So, 2, 0, 4.8, and π are constants. The constant π is approximately equal to 3.14.

Definition Constant

A **constant** is a symbol that represents a specific number (a quantity that does *not* vary).

Figure 1 One square inch

In the next example, we will compare the meanings of a variable and a constant while we consider the widths, lengths, and areas of some rectangles. The **area** (in square inches) of a flat surface is the number of square inches that it takes to cover the surface (see Fig. 1). The area of a rectangle is equal to the rectangle's length times its width.

Example 3 Comparing Constants and Variables

A rectangle has an area of 12 square inches. Let W be the width (in inches), L be the length (in inches), and A be the area (in square inches).

1. Sketch three possible rectangles of area 12 square inches.
2. Which of the symbols W, L, and A are variables? Explain.
3. Which of the symbols W, L, and A are constants? Explain.

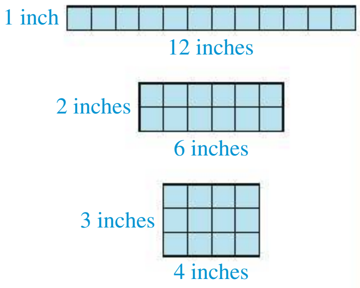

Figure 2 Three possible rectangles of area 12 square inches

Solution

1. We sketch three rectangles for which the width times the length is equal to 12 square inches (see Fig. 2).
2. The symbols W and L are variables, since they represent quantities that vary.
3. The symbol A is a constant, because in this problem the area does not vary—the area is always 12 square inches.

Counting Numbers

When we describe people, it often helps to describe them in terms of certain categories, such as gender, ethnicity, and employment. In mathematics, it helps to describe numbers in terms of categories, too. We begin by describing the *counting numbers,* which are the numbers 1, 2, 3, 4, 5, and so on.

Definition Counting numbers (natural numbers)

The **counting numbers,** or **natural numbers,** are the numbers

$$1, 2, 3, 4, 5, \ldots$$

The three dots mean that the pattern of the numbers shown continues without ending. In this case, the pattern continues with 6, 7, 8, and so on. When a list of numbers goes on forever, we say that there are an *infinite* number of numbers.

Integers

Next, we describe the *integers,* which include the counting numbers and other numbers.

Definition Integers

The **integers** are the numbers

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

The three dots on both sides mean that the pattern of the numbers shown continues without ending in both directions. In this case, the pattern continues with $-4, -5, -6$, and so on, and with 4, 5, 6, and so on.

The **positive integers** are the numbers $1, 2, 3, \ldots$. The **negative integers** are the numbers $-1, -2, -3, \ldots$. The integer 0 is neither positive nor negative. So, the integers consist of the counting numbers (which are positive integers), the negative integers, and 0.

The Number Line

We can visualize numbers on a *number line* (see Fig. 3).

Each point (location) on the number line represents a number. The numbers increase from left to right. We refer to the distance between two consecutive integers on the number line as 1 *unit* (see Fig. 3).

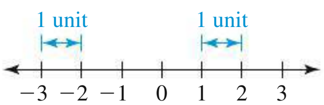

Figure 3 The number line